# MAT 303 Module Three Problem Set Report

Second Order Models

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**1. Introduction**

For this analysis, I have been provided with historical data related to wage growth and the labor force. The major goal is to determine how changes in these economic factors affect wage growth. We want to get a better knowledge of the dynamics of wage growth, unemployment, economy, and GDP growth, therefore giving useful insights for making economic decisions. I’ll be building a model using two quantitative variables and analyzing the findings. In the end, I will do a full second-order regression analysis with both quantitative and qualitative variables, broadening the scope of my analysis.

## **2. Data Preparation**

This data set includes the following variables: GDP growth rate, unemployment rate, inflation rate, wage growth rate, and whether the economy is experiencing a recession. The other factors serve as predictors, while the response variable is the wage growth rate. There are six columns and 99 rows in this data collection, with each row representing a different data sample.

## **3. Quadratic (Second Order) Model with One Quantitative Variable**

### Correlation Analysis

A graph of a growth curve

Description automatically generated with medium confidence

We can see that there is a nonlinear relationship between wage growth and unemployment in the scatterplot. Although there is an indication for wage growth to increase in parallel with unemployment, the slope curve indicates that this relationship is not linear, therefore a first order model would not be suitable in this situation. Given that the observed line is quadratic, a second order model would be the most appropriate fit to produce an accurate model.

### Reporting Results

E(y) = β0+ β1 x+ β2 x^2 is the generic form of the second-order regression model for wage growth with unemployment as the predictor variable, where x stands for unemployment and y for pay growth.

Based on the output, the prediction equation is y = 12.234 - 1.743 \* unemployment + 0.067 \* unemployment.

With an R-squared value of 0.9436, the model explains around 94.36% of the variation in wage growth. This indicates a strong fit, indicating that a substantial amount of the variance in wage increase is captured by the model. Even after accounting for the number of variables in the model, the Adjusted R-squared value of 0.9424 still demonstrates a strong explanatory power of 94.24%. This attests to the model's stability and lack of complexity.

In terms of beta estimates, the expected wage increase at zero unemployment is represented by the intercept, which is 12.2342. The unemployment coefficient is -1.7432. This negative figure shows that wage growth first declines when unemployment rises. Unemployment squared has a coefficient of 0.0674. This positive number indicates a parabolic relationship, whereby the pace of decline in wage growth slows down with increasing unemployment levels.

### Evaluating Model Significance

At the 5% level of significance, we use an overall F-test to assess the significance of the second-order regression model for wage growth with unemployment as the predictor variable. Assuming that all coefficients other than the intercept are equal to zero, the null hypothesis asserts that the model without predictors fits the data as well as the present model. According to the alternative hypothesis, the response variable has a substantial relationship with at least one of the predictors.

Less than 2.2𝑒−16 is the reported p-value for the F-test, which is significantly less than the 0.05 significance level. As a result, we determine that the model is significant at the 5% level of significance and reject the null hypothesis. This suggests that there is a substantial relationship between wage growth and at least one of the indicators (unemployment or unemployment squared).

Next, we use T-tests at a 5% level of significance to look at the importance of each term in the model. The intercept is significant as its p-value is less than 2e-16. Likewise, both the unemployment term (0.2869) and the unemployment squared term (-0.0146) have p-values below 2e-16, suggesting that they are similarly significant. Therefore, at the 5% level of significance, every term in the model is significant. Accordingly, both the linear and quadratic components of unemployment are significant predictors of wage growth. The positive coefficient for unemployment implies that wage growth will initially increase as unemployment rises, while the negative coefficient for unemployment squared shows that this positive relationship will eventually diminish and form a parabolic relationship at higher unemployment levels.

### Making Predictions Using Model

With the given unemployment rate of 2.54 and the regression equation, the expected wage growth is 8.2414. The calculated 95% prediction interval for wage growth is [6.9071, 9.5758]. We have a 95% confidence level that wage growth will take place within this range if the unemployment rate is 2.54. It provides a range of likely values for the wage growth prediction. The 95% confidence interval for the calculated wage growth is [8.0936, 8.3893]. This range shows that, with an unemployment rate of 2.54, we have a 95% confidence level that the average pay increase will fall inside it. An estimate of the mean response is given.

## **4. Complete Second Order Model with Two Quantitative Variables**

### Reporting Results

With GDP growth and unemployment as predictor variables, the second-order regression model for wage growth is E(y) = β0 + β1 x1 + β2 x2 + β3 x1x2 + β4 x2/1 + β5 x2/2.

According to the wage growth outputs, the prediction model equation is y = 8.989 − 1.153 \* unemployment + 0.284 \* GDP + 0.038 \* unemployment \* GDP − 0.0066 ⋅ unemployment^2 − 0.0063 ⋅ GDP^2.

With an R-squared value of 0.9587, the model explains around 95.87% of the variation in wage growth. This indicates a very good fit, indicating that a substantial amount of the variance in wage increase is captured by the model. Even after accounting for the number of predictors in the model, the Adjusted R-squared value of 0.9565 still demonstrates a strong explanatory power of 95.65%. This attests to the model's stability and lack of complexity.

According to beta estimates, GDP squared has a coefficient of -0.0066. This negative result implies a diminishing return impact, meaning that as GDP growth rises, the rate of wage growth slows down. The unemployment squared coefficient is 0.0377. This positive number shows that the detrimental effect on wage growth decreases as unemployment rises, creating a parabolic connection.

### Evaluating Model Significance

The overall F-test is the first tool we use to assess the regression model's model significance. According to the null hypothesis, all the regression model's coefficients are zero, indicating that it lacks explanatory power. According to the alternative hypothesis, there is at least one non-zero coefficient in the regression model. According to the F-test, the p-value is less than 2.2e - 16. We reject the null hypothesis since the p-value is significantly less than 0.05, indicating that the model is significant at the 5% level of significance. This suggests a substantial relationship between wage increase and at least one of the variables.

Let's now use T-tests at a 5% level of significance to investigate the importance of each term in the model. With a p-value of <2e-16 and an estimate of 8.9894, the intercept is significant. With a p-value of 8.26e-06 and an estimate of -1.1528, the unemployment term is likewise significant. Additionally important is the GDP term's estimate of 0.2837 and p-value of 0.04682. With a p-value of 0.00489 and an estimate of 0.0377, the unemployment squared term is significant. With a p-value of 0.12815 and an estimate of -0.0066, the GDP squared term is not significant. Likewise, the interaction term between GDP and unemployment has a p-value of 0.76678 and an estimate of -0.0063, both of which are not significant.

At the 5% threshold of significance, the model is deemed significant. Unemployment, GDP, the intercept, and unemployment squared all have significant individual terms, while GDP squared and the relationship between GDP and unemployment are not.

### Making Predictions Using Model

When unemployment is 2.50 and GDP growth is 6.50, the output predicts wage growth of 7.806 per year. The prediction interval indicates that we may have a 95% confidence level that the actual wage increase will lie between [6.6315, 8.9805]. Based on the stated GDP growth and unemployment rates, this interval offers a range of probable wage growth values. The 95% confidence interval for wage growth is [7.583, 8.0289], which indicates that we have a 95% chance that the average wage growth figure will fall inside this narrower range.

## **5. Complete Second Order Model with One Quantitative and One Qualitative Variable**

### Reporting Results

With unemployment and economy as predictor variables, the second-order regression model for wage growth is E(y) = β0 + β1 x1 + β2 x2 + β3 x1x2 + β4 x2/1 + β5 x2/1 x2.

According to the wage growth outputs, the prediction model equation is y = 12.36072 - 1.80834 \* unemployment - 2.70404 \* economy + 0.07574 \* unemployment^2 + 0.69359 \* (unemployment \* economy) - 0.04358 \* economy^2.

The R-squared value for the model is 0.9475, meaning that it can account for around 94.75% of the variation in wage growth. This illustrates how closely the response variable and the predictors are related. There is a 0.9446 Adjusted R-squared value. Even when the number of variables in the model is taken into consideration, this statistic indicates that the model is highly successful in predicting the variation in wage growth.

### Evaluating Model Significance

We can apply an overall F-test to assess the regression model's significance at a 5% level of significance. According to the F-test's null hypothesis, the model has no explanatory power if all regression coefficients are equal to zero. According to the alternative hypothesis, the model is significant if at least one regression coefficient is not equal to zero. According to the results, the F-test's P-value is less than 2.2e-16, which is significantly less than the significance level of 0.05. As a result, we determine that the model is significant at the 5% level and reject the null hypothesis.

Examining the individual terms, we find that both the economy-recession value and our unemployment p-value are 2e-16. After that, we have a squared unemployment of 0.0142. The ratio of unemployment to economic recession is 0.0272. These words are noteworthy since they are all below the 5% level of significance. However, we accept the null hypothesis since the P-value for the economy \* unemployment2 coefficient is 0.0512.

### Making Predictions Using Model

When unemployment is 2.50 and the economy is not in a recession, the expected wage growth is 8.3132. For a particular collection of predictor factors, this indicates that we have a 95% confidence level that the genuine wage growth will fall within this range. The 95% confidence interval, on the other hand, is [8.1573, 8.4692]. This interval is used to determine the range that, with 95% confidence, the true population mean of pay growth is most likely to lie inside. The prediction interval is greater than the confidence interval when accounting for the variability in individual wage growth data as well as the uncertainty in forecasting the mean wage growth.

## **6. Conclusion**

Following the analysis, this model seems to be a good fit for utilization, provided that the sample size is large enough. The high R-squared and adjusted R-squared values indicate that a substantial amount of the variance in wage increase is successfully captured. Significant coefficients for the economy, unemployment, and their interactions further highlight how crucial these determinants are in understanding changes in wage growth.

The results show that unemployment rates and economic conditions—whether there is a recession—have a significant impact on wage growth. The interaction terms show that whether the economy is in a recession or not affects how unemployment affects wage growth. Furthermore, a nonlinear connection is indicated by the quadratic factors, demonstrating that the impact of unemployment on wage growth varies at different unemployment levels.

Policymakers and economists will find these insights especially useful as they provide insight into the variables influencing wage growth. By comprehending these processes, policymakers may create focused economic policies, such steps to reduce unemployment, that could have a favorable effect on wage growth in times of economic stability. Additionally, this model provides useful information for decision-making in the public and commercial sectors by forecasting wage increases across a range of economic scenarios.